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Phonon emission from a disordered two-dimensional electron gas in a strong magnetic field

Keith A Benedict
Department of Physics, The University of Nottingham, Nottingham NG7 2RD, UK

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Abstract. The angular distribution of the phonon emission from a heated two-dimensional electron gas in a strong magnetic field is studied. It is shown that for processes involving inter-Landau level transitions a simple decoupling approximation is sufficient in treating the disorder; intra-Landau level processes are, however, seen to require more careful treatment and an analysis using the self-consistent Born approximation is given.

1. Introduction

Recent experiments [1] have used imaging techniques to study the angular distribution of phonons emitted by a two-dimensional electron gas (2DEG) which is heated relative to the lattice in which it is embedded by the passage of an electric current. The 2DEG gains energy from the applied field at a rate $\sigma_L(T_e)E^2$ where E is the applied field and $\sigma_L(T_e)$ is the dissipative part of the conductivity at temperature T_e. It is assumed that the electronic system rapidly equilibrates via the electron–electron (Coulomb) interaction in order that the effective electron temperature be meaningful. The 2DEG loses energy by emitting phonons, the rate of emission into the mode with polarization s and wavevector Q being $\tau_s^{-1}(Q;T_e,T_l)$ where T_l is the lattice temperature. Neglecting the warming of the lattice, the steady state electron temperature is determined by the condition

$$\sigma_L(T_e)E^2 = \sum_{s,Q} \hbar\omega_s(Q)\tau_s^{-1}(Q;T_e,T_l). \tag{1.1}$$

In silicon phonons travel ballistically for frequencies less than about 1THz, the critical frequency for isotope scattering to become relevant, until they reach the crystal surface; the imaging technique [1] assumes that the local temperature of a surface element $d\mathcal{S}(\theta,\phi)$ is proportional to the thermal flux incident on it, hence measurement of the temperature variation across the bottom of a sample provides a map of the angular distribution of the heat flux. Because of the elastic anisotropy of the lattice the energy current due to a phonon is not, in general, parallel to its wavevector: energy transport is directed along the group velocity of the phonon mode, i.e. along a normal to the surface of constant frequency in reciprocal space. Because the constant frequency surface for silicon is far from spherical, having regions which are concave, the phenomena of phonon focusing occurs [2] in which the energy flux is concentrated along

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the symmetry directions of the lattice. This effect can be seen [1] by injecting phonons isotropically (in Q space) from a 3D metal heater on the top surface of a silicon device with no 2DEG and imaging the energy flux on the bottom surface. Hence the effects of phonon focusing can, in principle, be deconvolved from the angular dependence of the emission from a 2DEG leaving the (angular) distribution of wavevectors of the emitted phonons which contains information on the electronic states. In all of the following we shall be attempting to calculate this Q-space distribution, the effects of phonon focusing will be neglected although they are required for direct comparison with experiment.

A very interesting case arises when the 2DEG is subject to a strong perpendicular magnetic field. Uchimura and Uemura [3] calculated the total emission in this case using the self-consistent Born approximation but only considered intra-Landau level processes. Toombs *et al* [4] have calculated the angular distribution of phonon emission in this case, assuming sharp Landau levels in the 2DEG. The macroscopic degeneracy of the states ensures that only phonons with integer multiples of the cyclotron frequency (cyclotron phonons) can be emitted, corresponding to inter-Landau level electronic transitions. In real devices the states are modified from this ideal case by the presence of disorder in the form of randomly placed ionized impurities and fluctuating potentials due to surface imperfections. The disorder broadens the Landau levels into bands and modifies the nature of the eigenstates, most of which become Anderson localized. The level broadening allows a range of phonon frequencies around each cyclotron harmonic as well as very low energy phonons due to intra-Landau level transitions.

It is not possible to treat the case of a disordered 2DEG exactly, so this paper presents calculations of the angular distribution of emitted phonons from a disordered 2DEG in a strong magnetic field in a sequence of three approximations. Firstly neglecting the disorder altogether which recovers the results of [4] in the present notation, secondly in a simple decoupling approximation which assumes an arbitrary form for the density of states of a Landau band but entirely neglects the correlations between the initial and final electronic states involved in a transition and finally using the self-consistent Born approximation (SCBA) of Ando *et al* [5] which treats two-particle correlations in a mean-field-like manner and shows true quantum diffusion; this approximation is exact in the limit of large Landau level index [6].

The SCBA is believed to provide an good description of the physics of the disordered 2DEG in a strong magnetic field on length scales of the order of the elastic scattering length, at longer length scales weak localization corrections begin to appear, while the macroscopic physics is described by the fixed points of the renormalization group flows which start from the values given by the SCBA [7]. In physical systems the scaling is cut off at the inelastic scattering length, this is relatively short at the electronic temperatures under discussion here and so the SCBA results should not be significantly renormalized and should be a sensible approximation for this problem.

The remainder of this paper is arranged as follows. In section 2 a simplified model of the experimental system is described which is the basis of subsequent calculations. Section 3 contains a derivation of the phonon emission rate and the angular distribution function in terms of the two-particle spectral function of the 2DEG. Section 4 contains the calculation of the distribution function in the absence of disorder, section 5 calculates it using the decoupling approximation while section 6 gives a derivation from the SCBA for Gaussian white-noise disorder, following [6]. Section 7 contains results from all three approximations for the case of emission from a silicon [100] 2DEG in the isotropic Debye approximation and section 8 is a brief summary.

2. The model system

In this section the simplified model of the experimental system will be described. A 2DEG of area Ω is embedded in an elastic medium of volume V. Directions in the medium are related to the normal to the 2DEG via angles θ, ϕ. The electronic motion is described by the single particle effective mass Hamiltonian ($\mathbf{r}$ and $\mathbf{p}$ are 2D vectors in the $z = 0$ plane)

$$\mathcal{H} = \mathcal{H}_0 + V(\boldsymbol{r}) = \frac{1}{2m^*}[\boldsymbol{p} - e\boldsymbol{A}(\boldsymbol{r})]^2 + V(\boldsymbol{r}) \tag{2.1}$$

where $\boldsymbol{A}(\boldsymbol{r})$ is the vector potential of a strong, uniform magnetic field, B, normal to the layer and $V(\boldsymbol{r})$ is a Gaussian white-noise distributed random potential satisfying

$$\begin{aligned} &\overline{V(\boldsymbol{r})} = 0 \\ &\overline{V(\boldsymbol{r})V(\boldsymbol{r}')} = 2\pi l_c^2 \lambda \delta(\boldsymbol{r} - \boldsymbol{r}') \end{aligned} \tag{2.2}$$

where the cyclotron length is $l_c = \sqrt{\hbar/eB}$ and $\lambda(B)$ is a measure of the disorder which is linear in the magnetic field. Motion of electrons perpendicular to the layer is assumed to be frozen out [20] so that the wavefunctions of all relevant electronic states have the form

$$\Psi_\alpha(\boldsymbol{r}, z) = \langle \boldsymbol{r}|\alpha\rangle f(z) \tag{2.3}$$

where $f(z)$ is a fixed function, characteristic of the 2DEG. In second quantized notation the Hamiltonian of the 2DEG is then

$$H_e = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha \tag{2.4}$$

where α labels the eigenstates of (2.1).

The Hamiltonian for the bulk phonon gas is simply

$$H_\phi = \sum_{s,\boldsymbol{Q}} \hbar\omega_s(\boldsymbol{Q}) a_s^\dagger(\boldsymbol{Q}) a_s(\boldsymbol{Q}) \tag{2.5}$$

where $\boldsymbol{Q}$ is a 3D wavevector with components $(\boldsymbol{q}, q_z)$. The electron–phonon coupling is assumed to be via the usual bulk deformation potential leading to an interaction of the form

$$H_{e\phi} = \sum_{\alpha,\beta,s,\boldsymbol{Q}} M_s^{\alpha,\beta}(\boldsymbol{Q}) c_\alpha^\dagger c_\beta (a_s(\boldsymbol{Q}) + a_s^\dagger(-\boldsymbol{Q})) \tag{2.6}$$

where the matrix element for phonon emission/absorption has the form

$$|M_s^{\alpha,\beta}(\boldsymbol{Q})|^2 = \frac{1}{V}\Lambda_s(\boldsymbol{Q})|\langle\alpha|e^{i\boldsymbol{q}\cdot\boldsymbol{r}}|\beta\rangle|^2. \tag{2.7}$$

$\Lambda_s(\boldsymbol{Q})$ is independent of the states of the 2DEG (although it does contain a form factor to account for the perpendicular part of the electronic wavefunction) an explicit form relevant to a [100] 2DEG in silicon will be given in section 7. The use of this form of interaction is an assumption, it is known that the presence of disorder does not modify the form of the electron-phonon coupling [8–10] but it is possible that the

two-dimensional nature of the electronic system entails a more complex coupling to the bulk lattice modes: at the very least there is strong evidence [11] that the value of the coupling constant relevant to 2D systems is markedly different to its value in bulk materials. The full Hamiltonian of the model thus has the form

$$H = H_e + H_\phi + H_{e\phi}. \tag{2.8}$$

A possible major shortcoming of this model is the neglect of the screening of the deformation potential by the electron gas, this is known to be important in bulk systems if quantitative predictions are required, but the nature of the screening in 2D systems is contentious and as no pretence of quantiative prediction is made here, the screening is neglected.

In all of the following it is assumed that the magnetic field is sufficiently strong that the Landau bands do not significantly overlap in energy, i.e. that

$$\sqrt{\lambda} \ll \hbar\omega_c \tag{2.9}$$

where $\omega_c = eB/m$ is the cyclotron frequency; this implies that the disorder does not significantly mix states from different Landau levels and hence that the Landau level index remains a good quantum number when the disorder is turned on.

3. The phonon emission rate and distribution function

At any instant the (mixed) quantum state of the system is described by a density matrix $\rho(t)$ [12], in the absence of the electron–phonon interaction this would have the form

$$\rho(t) = \rho_0 = \frac{e^{-(H_e-\mu N)/T_e}}{Z_e(T_e)} \frac{e^{-H_\phi/T_l}}{Z_\phi(T_l)} \tag{3.1}$$

where T_e and T_l are the electron and lattice temperatures and μ is the chemical potential of the 2DEG. It is mathematically convenient to assume that the system was in the state described by ρ_0 in the distant past and that the electron-phonon coupling was switched on adiabatically. Transforming to the interaction representation for the second quantized operators yields the following form for the von Neumann equation

$$i\hbar\partial_t\rho(t) = [H_{e\phi}(t), \rho(t)] : \qquad \rho(-\infty) = \rho_0. \tag{3.2}$$

This can be formally integrated to give a power series in the interaction for ρ. The mean occupation number of phonon mode $(s, \boldsymbol{Q})$ is

$$\nu_s(\boldsymbol{Q}, t) = \mathrm{Tr}\{\rho(t) n_s(\boldsymbol{Q})\} \tag{3.3}$$

where $n_s(\boldsymbol{Q}) = a_s^\dagger(\boldsymbol{Q}) a_s(\boldsymbol{Q})$ is the phonon number operator; hence the emission rate is, to second order in the interaction

$$\tau_s^{-1}(\boldsymbol{Q}) = \frac{d\nu_s(\boldsymbol{Q})}{dt} = -\frac{1}{\hbar^2}\int_0^\infty dt \left\langle [[n_s(\boldsymbol{Q}), H_{e\phi}(0)], H_{e\phi}(-t)] \right\rangle_0 + \ldots \tag{3.4}$$

where $\langle O(t)\rangle_0 = \mathrm{Tr}\{O(t)\rho_0\}$. The expectation value in (3.4) is taken with both systems separately in equilibrium and is straightforward, if tedious, to compute; it gives the following form for the emission rate

$$\tau_s^{-1}(Q) = \frac{2\pi}{\hbar}\sum_{\alpha,\beta}|M_s^{\alpha,\beta}(Q)|^2\delta(\epsilon_\alpha - \epsilon_\beta - \hbar\omega_s(Q)) \times f(\epsilon_\alpha)(1-f(\epsilon_\beta))C(\hbar\omega_s(Q);T_l,T_e) \tag{3.5}$$

where the detailed balance factor is

$$C(\omega;T_l,T_e) = \frac{e^{\omega/T_l} - e^{\omega/T_e}}{e^{\omega/T_l} - 1}. \tag{3.6}$$

Expression (3.5) is of course that which would be arrived at by the use of Fermi's golden rule assuming separate equilibria prior to each transition.

The total power emitted per unit area of the 2DEG is

$$r(T_e,T_l) = \frac{1}{\Omega}\sum_{s,Q}\frac{\hbar\omega_s(Q)}{\tau_s(Q)} \tag{3.7}$$

using the form (2.7) for the matrix element gives

$$r = \frac{2\pi}{V}\sum_{s,Q}\Phi_s(Q)\Gamma(\omega_s(Q),q) \tag{3.8}$$

where $\Phi_s(Q) = \omega_s(Q)\Lambda_s(Q)C(\hbar\omega_s(Q);T_l,T_e)$ and the structure function of the 2DEG is

$$\Gamma(\omega,q) = \sum_{\alpha,\beta}|\langle\alpha|e^{iq\cdot r}|\beta\rangle|^2\delta(\epsilon_\alpha - \epsilon_\beta - \hbar\omega_s(Q))f(\epsilon_\alpha)(1-f(\epsilon_\beta)) \tag{3.9}$$

where $f(E) = \{e^{(E-\mu)/T_e}+1\}^{-1}$ is the usual Fermi distribution. Taking the infinite volume limit gives

$$r = \int\frac{d^3Q}{4\pi^2}\sum_s P_s(Q) \tag{3.10}$$

where

$$P_s(Q) = P_s(Q,\theta,\phi) = \Phi_s(Q)\Gamma(\omega_s(Q),q) \tag{3.11}$$

from which we can define the experimentally relevant quantity, the phonon angular distribution function

$$W_s(\theta,\phi) = \int_0^\infty\frac{dQ}{4\pi^2}Q^2P_s(Q,\theta,\phi). \tag{3.12}$$

In terms of this we can write the total emitted power as

$$r(T_e,T_l) = \sum_s\int dS(\theta,\phi)\,W_s(\theta,\phi). \tag{3.13}$$

All of the information on the disordered electronic system is contained in the structure factor Γ; if we assume that the 2DEG is homogeneous and that Γ is a self-averaging quantity then, using the identity $\int \mathrm{d}E\,\delta(E-\epsilon_\alpha)=1$ in the definition (3.9), gives the following expression

$$\Gamma(\omega,q)=\int \mathrm{d}E\; f(E+\hbar\omega)\big(1-f(E)\big)S(q;E,\omega) \tag{3.14}$$

where the two particle spectral function [13]

$$\begin{aligned} S(q;E,\omega) &= \frac{1}{\Omega}\overline{\mathrm{tr}\big\{\delta(E+\hbar\omega-\mathcal{H})\mathrm{e}^{\mathrm{i}q\cdot r}\delta(E-\mathcal{H})\mathrm{e}^{-\mathrm{i}q\cdot r}\big\}} \\ &= \frac{1}{\Omega}\int \mathrm{d}^2r\;\mathrm{d}^2r'\;\mathrm{e}^{\mathrm{i}q\cdot(r-r')}\overline{\langle r|\delta(E+\hbar\omega-\mathcal{H})|r'\rangle}\;\overline{\langle r'|\delta(E-\mathcal{H})|r\rangle} \end{aligned} \tag{3.15}$$

has been introduced, which is commonly used in the theory of quantum transport.

4. Phonon emission from a pure 2DEG

In order to illustrate the above in an exactly solvable situation and to allow comparison with [4] the case of no disorder will be examined first. In all the following magnetic units will be used in which $\hbar=l_c=\omega_c=1$. In the Landau gauge the eigenstates of the single particle Hamiltonian

$$\mathcal{H}_0=\frac{1}{2m}\big(p-eA(r)\big)^2 \tag{4.1}$$

have the form

$$\langle r=(x,y)|n,k\rangle = C_n\mathrm{e}^{\mathrm{i}ky}\chi_n(x-k) \tag{4.2}$$

where $\mathcal{H}_0|n,k\rangle=\epsilon_n^0|n,k\rangle$ with $\epsilon_n^0=(n+1/2)$ and $\chi_n(x)$ is the nth harmonic oscillator wavefunction. Expanding the trace in equation (3.14) in terms of these basis states gives

$$S_0(q;E,\omega)=\frac{1}{\Omega}\sum_{n,n'}\delta(E+\omega-\epsilon_n^0)\delta(E-\epsilon_{n'}^0)\int \mathrm{d}^2r\;\mathrm{d}^2r'\;\mathrm{e}^{\mathrm{i}q\cdot(r-r')}C_n(r,r')C_{n'}(r',r) \tag{4.2}$$

where $C_n(r,r')=\sum_k\langle r|n,k\rangle\langle n,k|r'\rangle$ is the kernel of the projection operator onto the nth Landau level. The position space integrations can be performed to give

$$S_0(q;E,\omega)=\sum_{n,m=0}^{\infty}\delta(E+\omega-\epsilon_{n+m}^0)\delta(E-\epsilon_n^0)\frac{1}{2\pi}U_{n,m}(q^2/2) \tag{4.3}$$

where

$$U_{n,m}(x)=\frac{n!}{(n+m)!}x^m\mathrm{e}^{-x}\big[L_n^m(x)\big]^2 \tag{4.4}$$

with L_n^m an associated Laguerre polynomial [14]. Consequently the structure factor has the form

$$\Gamma_0(\omega, q) = \frac{1}{2\pi} \sum_m \delta(\omega - m) \sum_n U_{n,m}(q^2/2) f(\epsilon^0_{n+m})(1 - f(\epsilon^0_n)). \quad (4.5)$$

Bearing in mind that $|\mathbf{q}| = Q \sin\theta$ this gives

$$P_s(Q, \theta, \phi) = \Phi_s(Q) \frac{1}{2\pi\hbar} \sum_m \delta(\omega_s(Q) - m\omega_c) \times \sum_n U_{n,m}((l_c Q \sin\theta)^2/2) f(\epsilon^0_{n+m})(1 - f(\epsilon^0_n)) \quad (4.6)$$

clearly showing that only cyclotron phonons can be emitted. The form of equation (4.3) is worthy of comment, for each allowed transition it is the product of three factors: a density of initial states, a density of final states (both of which are delta functions at the relevant Landau energies) and a q dependent matrix element, this form will be seen to recurr in the approximation used in the next section.

5. Phonon emission in the decoupled approximation

It is not possible to evaluate (3.14) exactly for a disordered system so some approximation must be made. The simplest approximation is to replace the two- particle average by a product of one-particle averages setting

$$S_{\mathrm{dec}}(q; E, \omega) = \frac{1}{\Omega} \int \mathrm{d}^2 r \ \mathrm{d}^2 r' \ \mathrm{e}^{\mathrm{i}q \cdot (r - r')} \overline{\langle r|\delta(E + \hbar\omega - \mathcal{H})|r'\rangle} \ \overline{\langle r'|\delta(E - \mathcal{H})|r\rangle}. \quad (5.1)$$

The delta functions can be expressed in terms of the imaginary part of the resolvent operator $(\xi - \mathcal{H})^{-1}$ close to the real ξ axis, hence the average of the matrix element can be expressed in terms of the one-particle Green function as follows

$$\overline{\langle r|\delta(E + \hbar\omega - \mathcal{H})|r'\rangle} = \frac{\mathrm{i}}{2\pi} \lim_{\eta \to o^+} [G(r, r'; E + \mathrm{i}\eta) - G(r, r'; E - i\eta)]. \quad (5.2)$$

However, the strong field limit and the translational and rotational invariance of the ensemble of random potentials imply that [15,16]

$$G(r, r'; \xi) = \sum_{n=0}^{\infty} g_n(\xi) C_n(r, r') \quad (5.3)$$

where C_n is the kernel introduced in the previous section. The fact that $C_n(r, r) = 1/2\pi$ implies, using the standard relation between G and the density of states [17] that

$$\frac{\mathrm{i}}{2\pi} \lim_{\eta \to 0+} (g_n(E + \mathrm{i}\eta) - g_n(E - \mathrm{i}\eta)) = 2\pi\rho_n(E) \quad (5.3')$$

where $\rho_n(E)$ is the density of states in the nth Landau level. Thus it can be seen that

$$S_{\mathrm{dec}}(q; E, \omega) = \sum_{n,n'} (2\pi)^2 \rho_n(E + \omega) \rho_{n'}(E) \frac{1}{\Omega} \int \mathrm{d}^2 r \ \mathrm{d}^2 r' \ \mathrm{e}^{\mathrm{i}q \cdot (r - r')} C_n(r, r') C_{n'}(r', r)$$

$$= \frac{1}{2\pi} \sum_{n,m} (2\pi)^2 \rho_{n+m}(E + \omega) \rho_n(E) U_{n,m}(q^2/2). \quad (5.4)$$

This clearly reduces to the pure case in the limit $2\pi\rho_n(E) \to \delta(E-\epsilon_n^0)$ which corresponds to $\lambda \to 0$. Hence the structure function has the form

$$\Gamma_{\rm dec}(\omega,q) = \frac{1}{2\pi}\sum_{n,m} U_{n,m}(q^2)/2) \int {\rm d}E\ f(E+\omega)(1-f(E))(2\pi)^2\rho_{n+m}(E+\omega)\rho_n(E). \tag{5.5}$$

It is clear that the only effect of the disorder in this approximation is to broaden the energy spectrum of the electron gas, what has been omitted are the correlations between the eigenfunctions of the initial and final states in the transition due to their being in the same random potential, it will turn out that this is only a serious omission (at least for the present case of disorder without long-range spatial correlations) when considering intra-Landau level processes.

6. Phonon emission in the self-consistent Born approximation

The definition of the spectral function (3.14) shows that it is related to the usual two-particle Green functions, in fact it can be written as

$$S(q;E,\omega) = \frac{1}{2\pi^2}\{K^{+-}(q;E,\omega) + K^{-+}(q;E,\omega) - K^{++}(q;E,\omega) - K^{--}(q;E,\omega)\} \tag{6.1}$$

where

$$K^{\sigma,\sigma'}(q;E,\omega) = \int {\rm d}^2r\ {\rm e}^{{\rm i}q\cdot r}\overline{\langle r|(E+\omega+{\rm i}\sigma 0-\mathcal{H})^{-1}|{\rm o}\rangle\langle{\rm o}|(E+{\rm i}\sigma' 0-\mathcal{H})^{-1}|r\rangle}. \tag{6.2}$$

The self-consistent Born approximation for the one- and two-particle Green functions can be expressed diagrammatically as in figure 1 or as

$$G(r,r';z) = G^0(r,r';z) + 2\pi\lambda\int {\rm d}^2r''\ G^0(r,r'';z)G(r'',r'';z)G(r'',r';z) \tag{6.3}$$

and

$$\begin{aligned} K^{\sigma,\sigma'}(q;E,\omega) &= K_0^{\sigma,\sigma'}(q;E,\omega) + 2\pi\lambda K_0^{\sigma,\sigma'}(q;E,\omega)K^{\sigma,\sigma'}(q;E,\omega) \\ &= \frac{K_0^{\sigma,\sigma'}(q;E,\omega)}{1-2\pi\lambda K_0^{\sigma,\sigma'}(q;E,\omega)} \end{aligned} \tag{6.4}$$

where

$$G^0(r,r';z) = \sum_n \frac{C_n(r,r')}{z-\epsilon_n^0} \tag{6.5}$$

is the Green function for the pure 2DEG and

$$K_0^{\sigma,\sigma'}(q;E,\omega) = \frac{1}{2\pi}\sum_{n,m} g_{n+m}(E+{\rm i}\sigma 0+\omega)g_n(E+{\rm i}\sigma' 0)U_{n,m}(q^2/2) \tag{6.6}$$

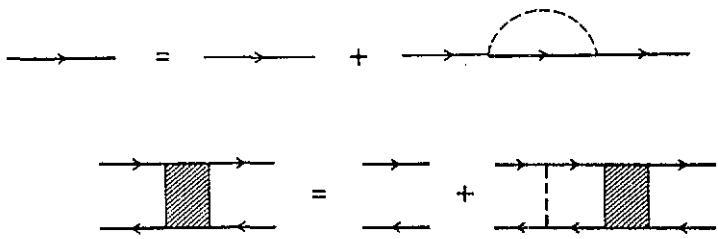

Figure 1. The diagrammatic forms of the Dyson and Bethe–Salpeter equations defining the SCBA.

(hence using K_0 instead of the full SCBA form gives the decoupled approximation).

Using relation (5.3) in the Dyson equation (6.3) reduces the latter to a simple quadratic equation for $g_n(z)$, the solution of which gives the density of states $\rho_n(E) = \rho(E - \epsilon_n^0)$ where

$$\rho(\eta) = \frac{1}{2\pi}\Theta(4\lambda - \eta^2)\frac{\sqrt{4\lambda - \eta^2}}{2\pi\lambda}. \tag{6.7}$$

The use of this approximation gives, on setting $E = \epsilon_n^0 + \delta\epsilon$ for some n and $\omega = m + \delta\omega$ for some m,

$$S(\boldsymbol{q}; \epsilon_n^0 + \delta\epsilon, m + \delta\omega) = \frac{u}{2\pi}\frac{(1-u^2)\rho(\delta\epsilon + \delta\omega)\rho(\delta\epsilon)}{(1-u^2)^2 - (u/\lambda)[\delta\epsilon(\delta\epsilon + \delta\omega)(1-u)^2 - u(\delta\omega)^2]} \tag{6.8}$$

where $u = U_{n,m}(q^2/2)$. It is not hard to show that this has the correct hydrodynamic limit ($m = 0; \delta\omega, q \to 0$) of

$$S(\boldsymbol{q}; E, \omega) = \frac{\rho(E)D(E)q^2/\pi}{\omega^2 + [D(E)q^2]^2} \tag{6.9}$$

which is characteristic of diffusive motion [18] with the diffusion constant $D(E) = 4\pi^2\lambda(n + 1/2)\rho(E)$. It is also clear that if u is small (e.g. when $q \to \infty$) then this reduces to the result of the decoupled approximation.

7. Results for [100] silicon

In this section results for the phonon distribution function and the total emitted power will be presented for the case of emission from a 2DEG formed at a [100] inversion layer in a silicon device. The isotropic Debye approximation is used for the phonon dispersion relation (only acoustic modes are considered), i.e.

$$\omega_s(\boldsymbol{Q}) = v_s Q \tag{7.1}$$

with $v_{\mathrm{LA}} = 9 \times 10^3 m\ s^{-1}$ and $v_{\mathrm{TA}} = 5.4 \times 10^3 m\ s^{-1}$. The phonon polarization vectors $\boldsymbol{P}^s(\boldsymbol{Q})$ are chosen so that $\boldsymbol{P}^{\mathrm{LA}}(\boldsymbol{Q}) = \boldsymbol{Q}/Q$ while the two TA modes are, respectively, in and perpendicular to the plane formed by $\boldsymbol{Q}$ and the normal to the 2DEG. The detailed form of the electron-phonon matrix element is taken to be [4,19]

$$M_s^{\alpha,\beta}(\boldsymbol{Q}) = \mathrm{i}\Xi\left(\frac{\hbar}{2\rho V\omega_s(\boldsymbol{Q})}\right)^{1/2}\left(q_z P_z^s(\boldsymbol{Q}) - D\boldsymbol{Q}\cdot\boldsymbol{P}^s(\boldsymbol{Q})\right) \times \int_0^\infty \mathrm{d}z\ f^*(z)\mathrm{e}^{\mathrm{i}q_z z} f(z)\langle\alpha|\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}|\beta\rangle. \tag{7.2}$$

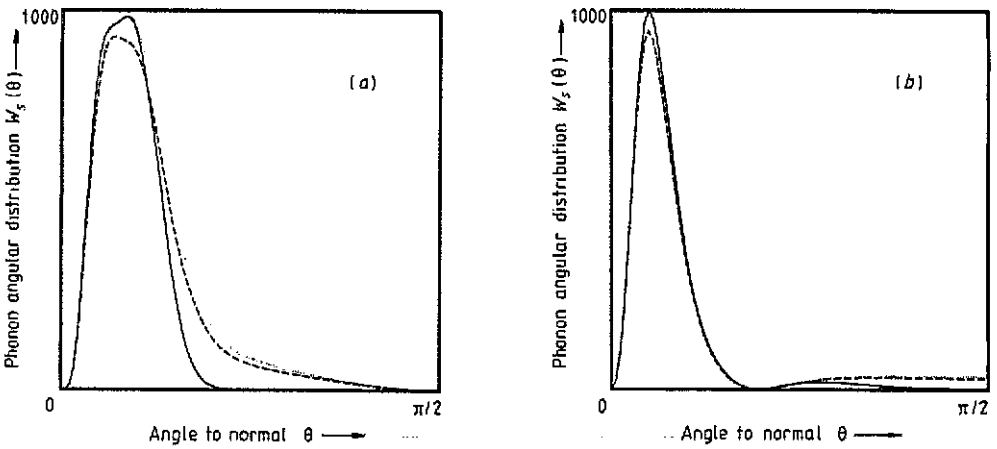

Figure 2. The angular distribution function $W(\theta)$ for (*a*) TA and (*b*) LA phonons at $B = 5$ T, $\mu = \hbar\omega_c$ and $k_B T_e = \frac{1}{2}\hbar\omega_c$.

The perpendicular part of the wavefunction is taken to have the standard variational form [20]

$$f(z) = (2a^3)^{-1/2} z e^{-z/2a}. \tag{7.3}$$

Hence, in the notation used in section 2

$$\Lambda_s(Q,\theta,\phi) = \frac{\hbar\Xi^2}{2\rho v_s Q} Q^2 \kappa_s(\theta) Z(q_z) \tag{7.4}$$

where $\kappa_{LA} = (\cos^2\theta - D)^2$, $\kappa_{TA1} = \cos^2\theta\sin^2\theta$ and $\kappa_{TA2} = 0$. The form factor for the thickness of the 2DEG is $Z(q_z) = (1 + a^2 q_z^2)^{-3}$. The parameters used in the results to be shown were: $D = -2/3$, $a = 1$nm (width of the 2DEG) and $m^* = 0.19m_e$, the absolute value of the emission rate is in consistent but rather arbitrary units (in fact they are W m^{-2} sr^{-1} assuming that the deformation constant has its bulk value of $\Xi = 9$ eV) comparison with experiment would proceed by normalizing $W_s(\theta)$ with respect to the measured peak value. In using the decoupled approximation the density of states function was chosen to be the elliptical band of the SCBA (equation (6.5)). All of the results shown correspond to a field of 5 T with the disorder parameter, λ, set such that the width of the disorder broadened Landau levels was one half of the cyclotron energy and the lattice temperature was set to zero. In each figure the full line is the result with no disorder, the dotted line gives the result of the decoupled approximation and the broken line is the SCBA result. Figure 2 shows the angular distribution of phonon emission in the three different approximations in the case where the chemical potential is placed half way between the $n = 0$ and $n = 1$ Landau levels at an electron temperature such that kT_e is one half of the cyclotron energy (approximately 17 K). Figure 3 shows the distributions for the case where the chemical potential is placed at the centre of the $n = 0$ Landau band at a temperature corresponding to one eighth of the cyclotron energy ($T_e = 4.4$ K). Figures 4 and 5 show the total emitted power as a function of electron temperature for the two choices of value of the chemical potential, while figures 6 and 7 show the variation of the total emitted power with chemical potential at the two values of electron temperature discussed above.

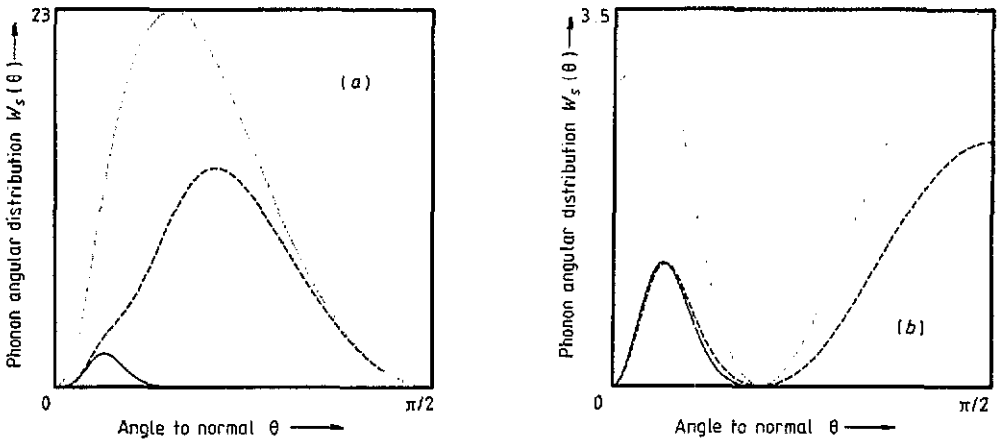

Figure 3. The angular distribution function $W(\theta)$ for (i) TA and (ii) LA phonons at $B = 5$ T, $\mu = \frac{1}{2}\hbar\omega_c$ and $k_B T_e = \frac{1}{8}\hbar\omega_c$.

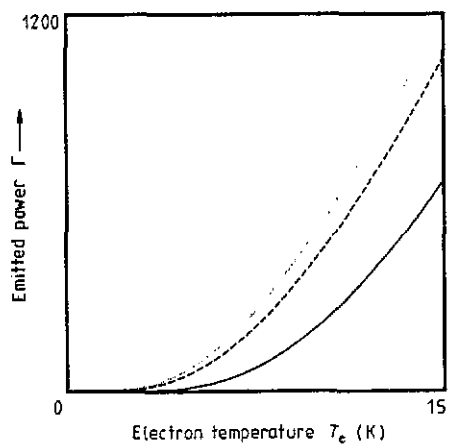

Figure 4. The total power emitted into TA modes as a function of electron temperature at $B = 5$ T and $\mu = \hbar\omega_c$.

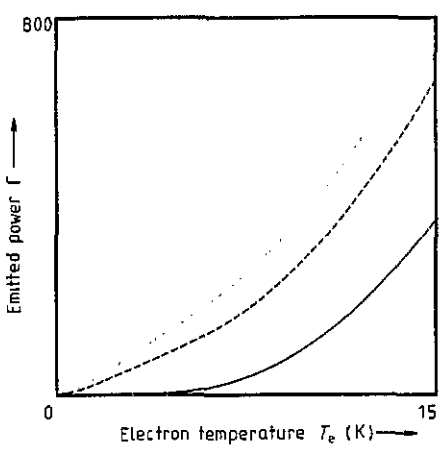

Figure 5. The total power emitted into TA modes as a function of electron temperature at $B = 5$ T and $\mu = \frac{1}{2}\hbar\omega_c$.

It is clear from these results that at higher temperatures where inter-Landau level transitions dominate the inclusion of disorder causes only quantitative modifications in the emission distribution, broadening it slightly, and that the difference in the results of the SCBA and the decoupled approximation are small: hence for most purposes at these higher temperatures the use of the decoupled approximation, with perhaps a more realistic choice of density of states function, will be sufficient. At the lower temperatures the intra-Landau level transitions will dominate, particularly when the Fermi level is within a Landau band, since these processes are forbidden in the absence of disorder, its inclusion makes a qualitative difference. Further, since the matrix element $U_{n,m}$ approaches 1 for some values of q when $m = 0$, the difference between the SCBA and the decoupled approximation is not negligible, hence in these circumstances the use of the results of section 6 will be required. It is possible that the structure in figure 7 could be used as the basis of a method using phonon emission to measure the

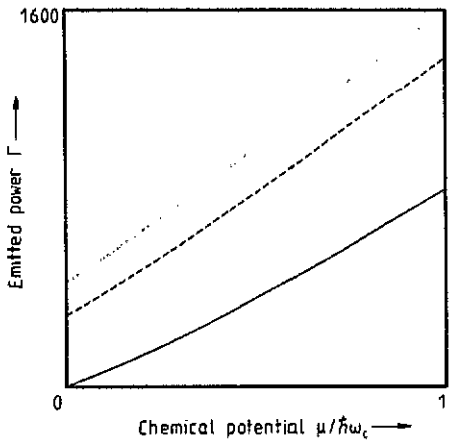

Figure 6. The total power emitted into TA modes as a function of chemical potential at $B = 5$ T and $k_B T_e = \frac{1}{2}\hbar\omega_c$.

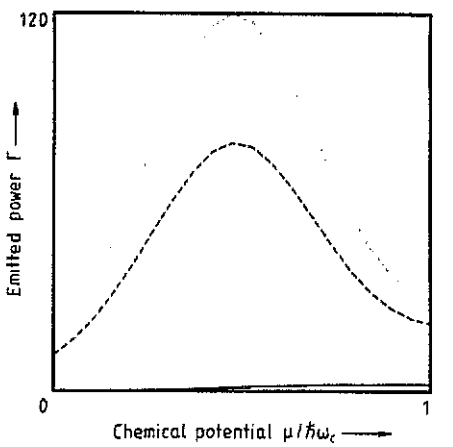

Figure 7. The total power emitted into TA modes as a function of chemical potential at $B = 5$ T and $k_B T_e = \frac{1}{8}\hbar\omega_c$.

density of states of a Landau band. The results for the total emission show that the decoupled approximation consistently overestimates the emitted power relative to the SCBA, this can be viewed as a reduction in the effective electron–phonon coupling due to the non-trivial correlations between the diffusive electronic states.

8. Summary

The distribution of emitted phonons from a heated disordered 2DEG in a strong magnetic field has been calculated using standard approximation techniques from the theory of disordered systems, it has been shown that at higher temperatures where inter-Landau level processes dominate a simple approximation using a suitable form for the density of states is sufficient while at lower temperatures where intra-Landau level processes dominate a more sophisticated treatment is needed.

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